DAY SIX

Work, Energy and Power

Learning & Revision for the Day

- Work
- Conservative and Non-conservative Force
- Energy
- Work-Energy Theorem
- Law of Conservation of Mechanical Energy
- Power
- Collision

Work

When a body is displaced actually through some distance in the direction of applied force, then work is said to be done by the force. The SI unit of work is joule (J) and in CGS is erg.

1 joule (J) $= 10^7$ erg

Work Done by a Constant Force

The work done by the force **F** in displacing the body through a distance **s** is

$$W = (F\cos\theta)s = Fs\cos\theta = \mathbf{F}\cdot\mathbf{s}$$

where, $F\cos\theta$ is the component of the force, acting along the direction of the displacement produced. SI unit of work is joule (J).

$$1\text{ J} = 1\text{ N-m}$$

Work is a scalar quantity. Work can be of three types

(i) Positive work (ii) Negative work and (iii) Zero work.

- **Positive work** If value of the angle θ between the directions of **F** and **s** is either zero or an acute angle.
- **Negative work** If value of angle θ between the directions of **F** and **s** is either 180° or an obtuse angle.
- As work done $W = \mathbf{F}\cdot\mathbf{s} = Fs\cos\theta$, hence **work done can be zero,** if

(i) No force is being applied on the body, i.e. $F = 0$.

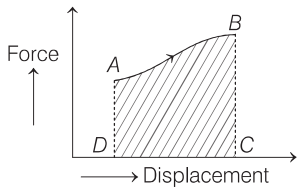
(ii) Although the force is being applied on a body but it is unable to cause any displacement in the body, i.e. $F \neq 0$ but $s = 0$.

(iii) Both F and s are finite but the angle θ between the directions of force and displacement is 90°. In such a case

$$W = \mathbf{F}\cdot\mathbf{s} = Fs\cos\theta = Fs\cos 90° = 0$$

Work Done by Variable Force

Work done by a variable force is given by $W = \int \mathbf{F} \cdot d\mathbf{s}$



It is equal to the area under the force-displacement graph, along with proper sign.

Work done = Area of $ABCDA$

Conservative and Non-Conservative Force

A force is said to be **conservative** if work done by or against the force in moving a body depends only on the initial and final positions of the body and not on the nature of path followed between the initial and the final position.

Gravitational force, force of gravity, electrostatic force are some examples of conservative forces (fields).

A force is said to be **non-conservative** if work done by or against the force in moving a body from one positions to another, depends on the path followed between these two positions. Force of friction and viscous force are the examples of non-conservative forces.

Energy

Energy is defined as the capacity or ability of a body to do work. Energy is scalar and its units and dimensions are the same as that of work. Thus, SI unit of energy is joule (J).

Some other commonly used units of energy are

$$1 \text{ erg} = 10^{-7} \text{ J}, \quad 1 \text{ cal} = 4.186 \text{ J} \simeq 4.2 \text{ J},$$

$$1 \text{ kcal} = 4186 \text{ J}, \; 1 \text{ kWh} = 3.6 \times 10^{6} \text{ J}$$

and $1 \text{ electron volt} = (1 \text{ eV}) = 1.60 \times 10^{-19} \text{ J}$

1. Kinetic Energy

Kinetic Energy (KE) is the capacity of a body to do work by virtue of its motion. A body of mass m, moving with a velocity v, has a kinetic energy, $K = \frac{1}{2} mv^2$.

Kinetic energy of a body is always positive irrespective of the sign of velocity v. Negative kinetic energy is impossible. Kinetic energy is correlated with momentum as

$$K = \frac{p^2}{2m}$$

or $$p = \sqrt{2mK}.$$

2. Potential Energy

Potential energy is the energy stored in a body or a system by virtue of its position in a field of force or due to its configuration. Potential energy is also called **mutual energy** or **energy of the configuration**.

Value of the potential energy in a given position can be defined only by assigning some arbitrary value to the reference point. Generally, reference point is taken at infinity and potential energy at infinity is taken as zero. In that case, $U = -W = -\int_{\infty}^{r} \mathbf{F} \cdot d\mathbf{r}$

Potential energy is a scalar quantity. It may be positive as well as negative.

Different types of potential energy are given below.

Gravitational Potential Energy

Gravitational potential energy of a body is the energy possessed by a body by virtue of its position above the surface of the earth.

i.e. Gravitational potential energy = mgh

where, m is the mass of a body at a height h above a reference level and g is acceleration due to gravity.

Potential Energy of a Spring

Whenever an elastic body (say a spring) is either stretched or compressed, work is being done against the elastic spring force. The work done is $W = \frac{1}{2} kx^2$,

where, k is spring constant and x is the displacement.

And elastic potential energy, $U = \frac{1}{2} kx^2$

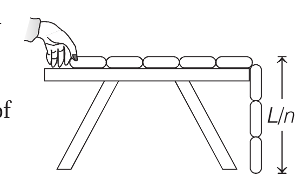
If spring is stretched from initial position x_1 to final position x_2, then

work done = Increment in elastic potential energy

$$= \frac{1}{2} k(x_2^2 - x_1^2).$$

Work Done in Pulling the Chain Against Gravity

If point mass m is pulled through a height h, then work done $W = mgh$.



For a chain, we can consider its centre of mass at the middle point of the hanging part i.e. at a height of $\left(\frac{L}{2n}\right)$ from the lower end and the mass of hanging part of chain, $m = \frac{M}{n}$.

So, workdone to raise the centre of mass of the chain on the table is given by

$$W = \frac{M}{n} \times g \times \frac{L}{2n} \Rightarrow W = \frac{MgL}{2n^2}$$

Work-Energy Theorem

Accordingly, work done by all the forces (conservative or non-conservative, external or internal) acting on a particle or an object is equal to the change in its kinetic energy of the particle. Thus, we can write $W = \Delta K = K_f - K_i$

We can also write, $K_f = K_i + W$

Which says that $\begin{pmatrix}\text{Kinetic energy after}\\ \text{the net work is done}\end{pmatrix}$

$$= \begin{pmatrix}\text{Kinetic energy before}\\ \text{the net work done}\end{pmatrix} + \begin{pmatrix}\text{The net}\\ \text{work done}\end{pmatrix}$$

Law of Conservation of Mechanical Energy

The mechanical energy E of a system is the sum of its kinetic energy K and its potential energy U.

$$E = K + U$$

When the forces acting on the system are conservative in nature, the mechanical energy of the system remains constant,

$$K + U = \text{constant} \Rightarrow \Delta K + \Delta U = 0$$

There are physical situations, where one or more non-conservative force act on the system but net work done by them is zero, then the mechanical energy of the system remains constant. If $\Sigma W_{\text{net}} = 0$

Mechanical energy, E = constant.

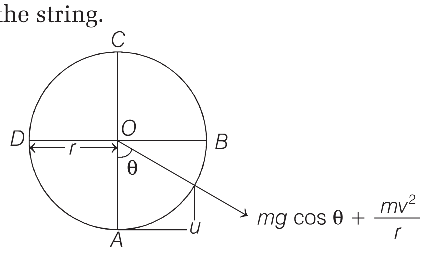
Motion in a Vertical Circle

In this motion body is under the influence of gravity of earth and total mechanical energy remains conserved
(K.E. converts into P.E and vice-versa).

- Velocity at any point on vertical loop

$$v = \sqrt{u^2 - 2g(1 - \cos\theta)r}$$

where, u is the initial velocity at lowest point and r is length of the string.



- Tension at any point on vertical loop

$$T - mg\cos\theta = \frac{mv^2}{r} \text{ or } T = \frac{m}{r}[u^2 - gr(2 - 3\cos\theta)]$$

Power

Power is defined as the rate of doing work. If an agent does work W in time t, then its average power is given by

$$P_{\text{av}} = \frac{W}{t}$$

The shorter is the time taken by a person or a machine in performing a particular task, the larger is the power of that person or machine.

Power is a scalar quantity and its SI unit is watt, where

$$1\,\text{W} = 1\ \text{J/s}$$

Instantaneous power, $\mathbf{P}_{\text{inst}} = \dfrac{dW}{dt} = \dfrac{\mathbf{F}\cdot d\mathbf{s}}{dt} = \mathbf{F}\cdot\mathbf{v}$

Some other commonly used units of power are

$$1\ \text{kW} = 10^3\ \text{W}, 1\ \text{MW} = 10^6\ \text{W and } 1\ \text{HP} = 746\ \text{W}.$$

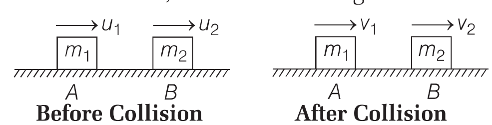
Collision

The physical interaction of two or more bodies in which each equal and opposite forces upon each others causing the exchange of energy and momentum is called collision.
Collisions are classified as

(i) elastic collisions and (ii) inelastic collisions.

1. Perfectly Elastic Collision in One Dimension

In a perfectly elastic collision, total energy and total linear momentum of colliding particles remain conserved. Moreover, the forces involved in interaction are conservative in nature and the total kinetic energy before and after the collision, remains unchanged.



In above figure, two bodies A and B of masses m_1 and m_2 and having initial velocities u_1 and u_2 in one dimension, collide elastically and after collision move with velocities v_1 and v_2, then we find that

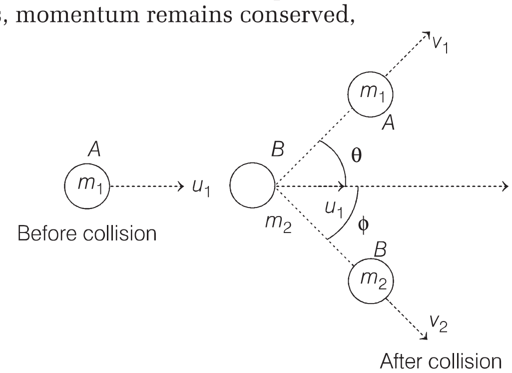
(i) Relative velocity of approach = Relative velocity of separation, i.e. $u_1 - u_2 = v_2 - v_1$

(ii) $$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2$$

and $$v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2$$

2. Perfectly Elastic Collision in a Plane

In a two dimensional (or oblique) collision between two bodies, momentum remains conserved,



∴ Along the X-axis

$$m_1 u_1 + m_2 u_2 = m_1 v_1 \cos\theta + m_2 v_2 \cos\phi \quad \ldots \text{(i)}$$

and along the Y-axis

$$0 = m_1 v_1 \sin\theta - m_2 v_2 \sin\phi \quad \ldots \text{(ii)}$$

As the total kinetic energy remains unchanged.

Hence, $$\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \quad \ldots \text{(iii)}$$

We can solve these equations provided that either the value of θ or ϕ is known to us.

3. Inelastic Collision

In an inelastic collision, the total linear momentum as well as total energy remain conserved but total kinetic energy after collision is not equal to kinetic energy before collision.

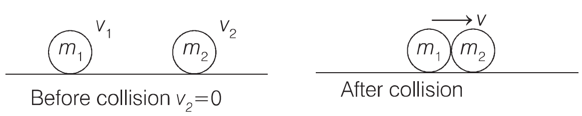
For the type of collision

Common speed $v = \dfrac{m_1 v_1}{m_1 + m_2}$

and loss of kinetic energy $\Delta K = \dfrac{m_1 m_2 (v_1 - v_2)^2}{2(m_1 + m_2)}$

Here $v_2 = 0$

∴ $$\Delta K = \frac{m_1 m_2 v_1^2}{2(m_1 + m_2)}$$



Coefficient of Restitution (e)

For a collision, it is defined as the ratio of relative velocity of separation to the relative velocity of approach.

Thus, coefficient of restitution $e = \dfrac{v_2 - v_1}{u_1 - u_2}$

For a perfectly elastic collision $(e) = 1$.

If $0 < e < 1$, the collision is said to be partially elastic. But if $e = 0$, then collision is said to be perfectly inelastic.

In a perfectly inelastic collision, $e = 0$ which means that $v_2 - v_1 = 0$ or $v_2 = v_1$.

It can be shown that for an inelastic collision the final velocities of the colliding bodies are given by

$$v_1 = \left(\frac{m_1 - em_2}{m_1 + m_2}\right) u_1 + \frac{(1+e)\, m_2}{(m_1 + m_2)} u_2$$

and $$v_2 = \frac{(1+e)\, m_1}{(m_1 + m_2)} u_1 + \left(\frac{m_2 - em_1}{m_1 + m_2}\right) u_2.$$

If a particle of mass m, moving with velocity u, hits an identical stationary target inelastically, then final velocities of projectile and target are correlated as

i.e. $m_1 = m_2 = m$ and $u_2 = 0$; $\dfrac{v_1}{v_2} = \dfrac{1-e}{1+e}$.

Rebounding of a Ball on Collision with the Floor

- Speed of the ball after the nth rebound

$$v_n = e^n v_0 = e^n \sqrt{2gh_0}$$

- Height covered by the ball after the nth rebound

$$h_n = e^{2n} h_0$$

- Total distance (vertical) covered by the ball before it stops bouncing

$$H = h_0 + 2h_1 + 2h_2 + 2h_3 + \ldots = h_0 \left(\frac{1+e^2}{1-e^2}\right)$$

- Total time taken by the ball before it stops bouncing

$$T = t_0 + t_1 + t_2 + t_3 + \ldots = \sqrt{\frac{2h_0}{g}} + 2\sqrt{\frac{2h_1}{g}} + 2\sqrt{\frac{2h_2}{g}} + \ldots$$

$$= \sqrt{\frac{2h_0}{g}} \left(\frac{1+e}{1-e}\right)$$

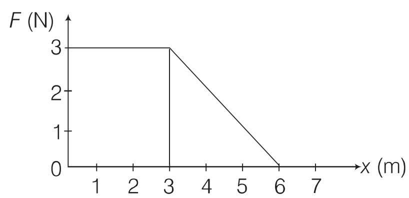
DAY PRACTICE SESSION 1

FOUNDATION QUESTIONS EXERCISE

1 A bicyclist comes to a skidding stop in 10 m. During this process, the force on the bicycle due to the road is 200 N and is directly opposed to the motion. The work done by the cycle on the road is

(a) +2000 J (b) –200 J (c) Zero (d) –20000 J

2 A force F acting on an object varies with distance x as shown here. The force is in newton and x is in metre. The work done by the force in moving the object from $x = 0$ to $x = 6$ m is

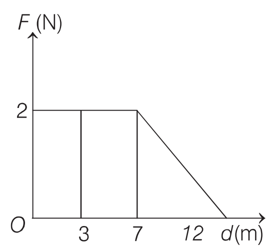


(a) 4.5 J (b) 13.5 J (c) 9.0 J (d) 18.0 J

3 A particle moves from a point $(-2\hat{\mathbf{i}} + 5\hat{\mathbf{j}})$ to $(4\hat{\mathbf{j}} + 3\hat{\mathbf{k}})$ when a force of $(4\hat{\mathbf{i}} + 3\hat{\mathbf{j}})$ N is applied. How much work has been done by the force? ➔ NEET 2016

(a) 8 J (b) 11 J (c) 5 J (d) 2 J

4 Force F on a particle moving in a straight line varies with distance d as shown in the figure. The work done on the particle during its displacement of 12 m is ➔ CBSE AIPMT 2011



(a) 21 J (b) 26 J
(c) 13 J (d) 18 J

5 A uniform force of $(3\hat{\mathbf{i}} + \hat{\mathbf{j}})$ N acts on a particle of mass 2 kg. Hence, the particle is displaced from position $(2\hat{\mathbf{i}} + \hat{\mathbf{k}})$ m to position $(4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}})$ m. The work done by the force on the particle is ➔ NEET 2013

(a) 9 J (b) 6 J (c) 13 J (d) 15 J

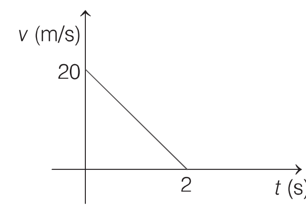
6 A particle moves along the X-axis from $x = 0$ to $x = 5$ m under the influence of a force given by $F = 7 - 2x + 3x^2$. The work done in the process is

(a) 70 J (b) 270 J
(c) 35 J (d) 135 J

7 The potential energy of a system increases, if work is done ➔ CBSE AIPMT 2011

(a) by the system against a conservative force
(b) by the system against a non-conservative force
(c) upon the system by a conservative force
(d) upon the system by a non-conservative force

8 Velocity-time graph of particle of mass 2 kg moving in a straight line is as shown in figure. Work done by all the forces on the particle is



(a) 400 J (b) – 400 J
(c) – 200J (d) 200J

9 A ball of mass 2 kg and another of mass 4 kg are dropped together from a 60 ft tall building. After a fall of 30 ft each towards the earth, their respective kinetic energies will be in the ratio of

(a) $\sqrt{2} : 1$ (b) 1: 4 (c) 1: 2 (d) $1 : \sqrt{2}$

10 The momentum of a body is increased by 20%. The percentage increase in kinetic energy is

(a) 54% (b) 44% (c) 100% (d) 50%

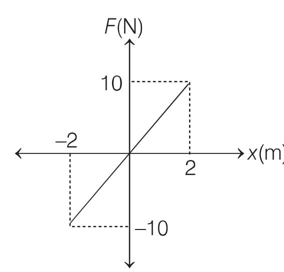
11 A body of mass m is accelerated uniformly from rest to a speed v in a time T. The instantaneous power delivered to the body as a function of time is, given by

(a) $\frac{mv^2}{T^2}t$ (b) $\frac{mv^2}{T^2}t^2$ (c) $\frac{1}{2}\frac{mv^2}{T^2}t$ (d) $\frac{1}{2}\frac{mv^2}{T^2}t^2$

12 The power supplied by a force acting on a particle moving in a straight line is constant. The velocity of the particle varies with the displacement x as

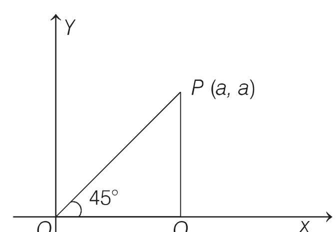
(a) $x^{1/2}$ (b) x
(c) x^2 (d) $x^{1/3}$

13 A force F acting on a particle varies with the position x as shown in the figure. The work done by this force in displacing the particle from $x = -2$ m to $x = 0$ is



(a) – 10 J (b) + 5 J
(c) – 20 J (d) + 40 J

14 A particle is moved from $(0, 0)$ to (a, a) under a force $\mathbf{F} = (3\hat{\mathbf{i}} + 4\hat{\mathbf{j}})$ from two paths. Path 1 is OP and path 2 is OQP. Let W_1 and W_2 be the work done by this force in these two paths. Then,



(a) $W_1 = W_2$ (b) $W_1 = 2W_2$
(c) $W_2 = 2W_1$ (d) $W_2 = 4W_1$

15 An engine pumps water continuously through a hose. Water leaves the hose with a velocity v and m is the mass per unit length of the water jet. What is the rate at which kinetic energy is imparted to water ?

(a) $\frac{1}{2}mv^3$ (b) mv^3 (c) $\frac{1}{2}mv^2$ (d) $\frac{1}{2}m^2v^2$

16 300 J of work is done in sliding a 2 kg block up an inclined plane of height 10 m. Taking $g = 10\ ms^{-2}$, work done against friction is

(a) 200 J (b) 100 J (c) zero (d) 1000 J

17 The potential energy of a particle in a force field is $U = \frac{A}{r^2} - \frac{B}{r}$, where A and B are positive constants and r is the distance of particle from the centre of the field. For stable equilibrium, the distance of the particle is ➔ CBSE AIPMT 2012

(a) $B/2A$ (b) $2A/B$ (c) A/B (d) B/A

18 A spring of spring constant $5 \times 10^3\ Nm^{-1}$ is stretched initially by 5 cm from the unstretched position. Then, the work required to stretch it further by another 5 cm is

(a) 12.50 N-m (b) 18.75 N-m (c) 25.00 N-m (d) 6.25 N-m

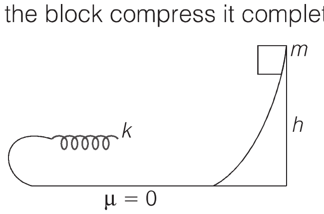
19 Two similar springs P and Q have spring constants k_P and k_Q, such that $k_P > k_Q$. They are stretched, first by the same amount (case a), then by the same force (case b). The work done by the springs W_P and W_Q are related as, in case (a) and case (b), respectively ➔ CBSE AIPMT 2015

(a) $W_P = W_Q; W_P > W_Q$ (b) $W_P = W_Q; W_P = W_Q$
(c) $W_P > W_Q; W_Q > W_P$ (d) $W_P < W_Q; W_Q < W_P$

20. When a bullet of mass 10 g and speed 100 ms^{-1} penetrates up to distance 1 cm in a human body in rest. The resistance offered by human body is

(a) 2000 N (b)1500 N (c) 5000 N (d) 1000 N

21 A block is left on a frictionless curve path that connects to a horizontal path. At the end of the horizontal path, there is a spring connected. Find the work done by spring, when the block compress it completely.



(a) $-\frac{mk}{h}$ (b) $-mgh$ (c) mgh (d) $\frac{mk}{h}$

22 Consider a drop of rain water having mass 1 g falling from a height of 1 km. It hits the ground with a speed of 50 m/s. Take g constant with a value of 10 m/s^2. The work done by the (i) gravitational force and the (ii) resistive force of air is ➔ NEET 2017

(a) (i) –10 J, (ii) –8.25 J (b) (i) 1.25 J, (ii) –8.25 J
(c) (i) 100 J, (ii) 8.75 J (d) (i) 10 J, (ii) – 8.75 J

23 A block of mass 10 kg, moving in x-direction with a constant speed of $10\ ms^{-1}$, is subjected to a retarding force $F = 0.1x$ J/m during its travel from $x = 20$ m to 30 m. Its final KE will be ➔ CBSE-AIPMT 2015

(a) 475 J (b) 450 J (c) 275 J (d) 250 J

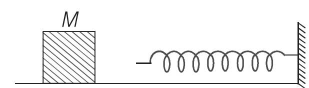
24 A small block of super dense material has mass 3×10^{24} kg. It is situated at a height $h\ (<< R_e)$ from where it falls on the earth's surface. Its speed when its height has reduced to $\frac{h}{2}$ is (given mass of the earth is 6×10^{24} kg)

(a) $\sqrt{\frac{2gh}{3}}$ (b) $\sqrt{\frac{3}{2}gh}$ (c) zero (d) $\sqrt{gh}$

25 A block of mass M is attached to the lower end of a vertical spring. The spring is hung from a ceiling and has force constant value k. The mass is released from rest with the spring initially unstretched. The maximum extension produced in the length of the spring will be ➔ CBSE AIPMT 2009

(a) Mg/k (b) $2\ Mg/k$ (c) $4\ Mg/k$ (d) $Mg/2k$

26 The block of mass M moving on the frictionless horizontal surface collides with the spring of spring constant k and compresses it by length L. The maximum momentum of the block after collision is



(a) $\sqrt{Mk}\,L$ (b) $\frac{kL^2}{2M}$ (c) zero (d) $\frac{ML^2}{k}$

27 A body of mass 1 kg is thrown upwards with a velocity 20 ms^{-1}. It momentarily comes to rest after attaining a height of 18 m. How much energy is lost due to air friction? (Take, $g = 10\ ms^{-2}$) ➔ CBSE AIPMT 2009

(a) 20 J (b) 30 J (c) 40 J (d) 10 J

28 The string of a pendulum is horizontal. The mass of bob attached to it is m. Now, the string is released. The tension in the string in the lowest position is

(a) mg (b) $2mg$ (c) $3mg$ (d) $4mg$

29 A stone is tied to a string of length l and is whirled in a vertical circle with the other end of the string as the centre. At a certain instant of time, the stone is at its lowest position and has a speed u. The magnitude of the change in velocity as it reaches a position where the string is horizontal (g being acceleration due to gravity) is

(a) $\sqrt{2(u^2 - gl)}$ (b) $\sqrt{u^2 - gl}$ (c) $v - \sqrt{u^2 - 2gl}$ (d) $\sqrt{2gl}$

30 A heavy stone hanging from a massless string of length 15 m is protected horizontally with speed 147 m/s. The speed of the particle at the point where tension in the string equals the weight of the particle is

(a) 10 m/s (b) 7 m/s
(c) 12 m/s (d) None of these

31 A stone of mass 1 kg is tied of a string 4m long and is rotated at constant speed of 40 m/s in a vertical circle. The ratio of the tension at the top and the bottom is (Take, $g = 10\ \text{m/s}^2$)

(a) 11 : 12 (b) 39 : 41 (c) 41 : 39 (d) 12 : 11

32 A train of mass 2×10^5 kg has a constant speed of 20 m/s up a hill inclined at $\theta = \sin^{-1}\left(\frac{1}{50}\right)$ to the horizontal when the engine is working at 8×10^5 W. The resistance to motion of train is

(a) 400 N (b) 200 N (c) 600 N (d) 800 N

33 A particle of mass m is driven by a machine that delivers a constant power k watts. If the particle starts from rest the force on the particle at time t is ➜ CBSE AIPMT 2015

(a) $\sqrt{\frac{mk}{2}}\, t^{-1/2}$ (b) $\sqrt{mk}\, t^{-1/2}$

(c) $\sqrt{2mk}\, t^{-1/2}$ (d) $\frac{1}{2}\sqrt{mk}\, t^{-1/2}$

34 An engine pumps water through a hose pipe. Water passes through the pipe and leaves it with a velocity of $2\ \text{ms}^{-1}$. The mass per unit length of water in the pipe is $100\ \text{kg m}^{-1}$. What is the power of the engine? ➜ CBSE AIPMT 2010

(a) 400 W (b) 200 W (c) 100 W (d) 800 W

35 Two particles of masses m_1 and m_2 move with initial velocities u_1 and u_2. On collision, one of the particles get excited to higher level, after absorbing energy E. If final velocities of particles be v_1 and v_2, then we must have ➜ CBSE AIPMT 2015

(a) $m_1^2u_1 + m_2^2u_2 - E = m_1^2v_1 + m_2^2v_2$

(b) $\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 - E$

c) $\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 - E = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

(d) $\frac{1}{2}m_1^2u_1^2 + \frac{1}{2}m_2^2u_2^2 + E = \frac{1}{2}m_1^2v_1^2 + \frac{1}{2}m_2^2v_2^2$

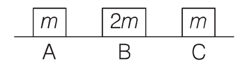
36 Two identical balls A and B having velocities of 0.5 m/s and – 0.3 m/s respectively, collide elastically in one dimension. The velocities of B and A after the collision respectively will be ➜ NEET 2016

(a) –0.5 m/s and 0.3 m/s (b) 0.5 m/s and –0.3 m/s
(c) –0.3 m/s and 0.5 m/s (d) 0.3 m/s and 0.5 m/s

37 An explosion breaks a rock into three parts in a horizontal plane. Two of them go off at right angles to each other. The first part of mass 1kg moves with a speed of $12\ \text{ms}^{-1}$ and the second part of mass 2 kg moves with $8\ \text{ms}^{-1}$ speed. If the third part flies off with $4\ \text{ms}^{-1}$ speed, then its mass is ➜ NEET 2013

(a) 3 kg (b) 5 kg
(c) 7 kg (d) 17 kg

38 Three objects A, B and C are kept in a straight line on a frictionless horizontal surface. These have masses m, $2m$ and m, respectively. The object A moves towards B with a speed 9 m/s and makes an elastic collision with it. Thereafter, B makes completely inelastic collision with C. All motions occur on the same straight line. Find the final speed (in m/s) of the object C.



(a) 3 m/s (b) 4 m/s (c) 5 m/s (d) 1 m/s

39 A particle A suffers on oblique elastic collision with a particle B, i.e. at rest initially. If their masses are the same, then after the collision

1. their KE may be equal
2. a continuous to move in the original direction while B remains at rest
3. they will move in mutually perpendicular directions
4. a comes to rest and B starts moving in the direction of the original motion of A

(a) 1, 3 (b) 2, 3 (c) 1, 2 (d) 1, 2, 3

40 A body of mass $(4m)$ is lying in xy-plane at rest. It suddenly explodes into three pieces. Two pieces each of mass m move perpendicular to each other with equal speeds v. The total kinetic energy generated due to explosion is ➜ CBSE AIPMT 2014

(a) mv^2 (b) $\frac{3}{2}mv^2$ (c) $2mv^2$ (d) $4mv^2$

41 For inelastic collision between two spherical rigid bodies

(a) the total kinetic energy is conserved
(b) the total mechanical energy is not conserved
(c) the linear momentum is not conserved
(d) the linear momentum is conserved

42 A moving neutron collides with a stationary α-particle. The fraction of the kinetic energy lost by the neutron is

(a) 16/25 (b) 9/25 (c) 3/5 (d) 2/5

43 A block of mass m moving at a velocity v collides with another block of mass $2m$ at rest. The lighter block comes to rest after collision. Find the coefficient of restitution.

(a) 1/2 (b) 1 (c) 1/3 (d) 1/4

44 A ball is thrown vertically downwards from a height of 20m with an initial velocity v_0. It collides with the ground, loses 50% of its energy in collision and rebounds to the same height. The initial velocity v_0 is (Take, $g = 10\ \text{ms}^{-2}$) ➜ CBSE AIPMT 2015

(a) $14\ \text{ms}^{-1}$ (b) $20\ \text{ms}^{-1}$
(c) $28\ \text{ms}^{-1}$ (d) $10\ \text{ms}^{-1}$

45 A bullet of mass m moving with velocity v strikes a block of mass M at rest and gets embedded into it. The kinetic energy of the composite block will be

(a) $\frac{1}{2}mv^2 \times \frac{m}{(m+M)}$ (b) $\frac{1}{2}mv^2 \times \frac{M}{(m+M)}$

(c) $\frac{1}{2}mv^2 \times \frac{(M+m)}{M}$ (d) $\frac{1}{2}Mv^2 \times \frac{m}{(m+M)}$

DAY PRACTICE SESSION 2

PROGRESSIVE QUESTIONS EXERCISE

1 A body of mass 10 kg is moving on an inclined plane of inclination 30° with an acceleration 2 ms^{-2}. The body starts from rest, the work done by force of gravity in 2 s is

(a) 10 J (b) zero (c) 98 J (d) 196 J

2 A body of mass 0.5 kg travels in a straight line with velocity $v = a\ x^{3/2}$, where $a = 5 ms^{-2}$. The work done by the net force during its displacement from $x = 0$ to $x = 2$ m is ➔ NCERT Exemplar

(a) 1.5 J (b) 50 J (c) 10 J (d) 100 J

3 A particle free to move along X-axis has potential energy given as $U(X) = k(1 - e^{-X^2})$, where k is a positive constant of appropriate dimension. Then, for $-a < X < \infty$

(a) at points away from the origin, the particle is in unstable equilibrium
(b) for any finite non-zero value of x, there is a force directed away from the origin
(c) if its total mechanical energy is $\frac{k}{2}$, it has its minimum kinetic energy at the origin
(d) if its total mechanical energy is $\frac{k}{2}$, it has its maximum kinetic energy at origin

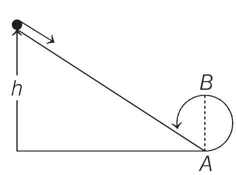
4. A particle of mass 10 g moves along a circle of radius 6.4 cm with a constant tangential acceleration. What is the magnitude of this acceleration, if the kinetic energy of the particle becomes equal to 8×10^{-4} J by the end of the second revolution after the beginning of the motion? ➔ NEET 2016

(a) 0.15 m/s^2 (b) 0.18 m/s^2
(c) 0.2 m/s^2 (d) 0.1 m/s^2

5 A block of mass 10 kg is moving in x-direction with a constant speed of 10 ms^{-1}. It is subjected to a retarding force $F = -0.1\ x\ Jm^{-1}$, during its travel from $x = 20$ m to $x = 30$ m. Its final kinetic energy will be

(a) 475 J (b) 450 J (c) 275 J (d) 250 J

6 A body initially at rest and sliding along a frictionless track from a height h (as shown in the figure) just completes a vertical circle of diameter $AB = D$. The height h is equal to ➔ NEET 2018



(a) $\frac{7}{5}D$ (b) D (c) $\frac{3}{2}D$ (d) $\frac{5}{4}D$

7 What is the minimum velocity with which a body of mass m must enter a vertical loop of radius R so that it can complete the loop? ➔ NEET 2016

(a) $\sqrt{2gR}$ (b) $\sqrt{3gR}$
(c) $\sqrt{5gR}$ (d) $\sqrt{gR}$

8 A body of mass 1 kg begins to move under the action of a time dependent force $\mathbf{F} = (2t\,\hat{\mathbf{i}} + 3t^2\,\hat{\mathbf{j}})$ N, where $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ are unit vectors along X and Y axes. What power will be developed by the force at the time (t)? ➔ NEET 2016

(a) $(2t^2 + 4t^4)$ W (b) $(2t^3 + 3t^4)$ W
(c) $(2t^3 + 3t^5)$ W (d) $(2t + 3t^3)$ W

9 On a frictionless surface, a block of mass M moving at speed v collides elastically with another block of same mass M which is initially at rest. After collision the first block moves at an angle θ to its initial direction and has a speed $\frac{v}{3}$. The second block's speed after the collision is

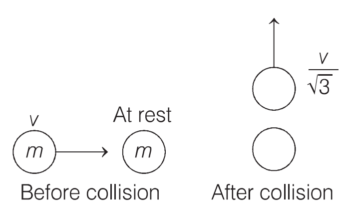
➔ CBSE AIPMT 2015

(a) $\frac{2\sqrt{2}}{3}v$ (b) $\frac{3}{4}v$
(c) $\frac{3}{\sqrt{2}}v$ (d) $\frac{\sqrt{3}}{2}v$

10 A moving block having mass m, collides with another stationary block having mass $4m$. The lighter block comes to rest after collision. When the initial velocity of the lighter block is v, then the value of coefficient of restitution (e) will be ➔ NEET 2018

(a) 0.8 (b) 0.25
(c) 0.5 (d) 0.4

11 A mass m moves with a velocity v and collides inelastically with another identical mass. After collision the first mass moves with velocity $\frac{v}{\sqrt{3}}$ in a direction perpendicular to the initial direction of motion. Find the speed of the second mass after collision.

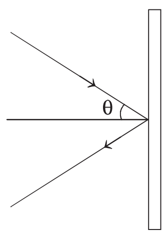


(a) $\frac{2}{\sqrt{3}}v$ (b) $\frac{v}{\sqrt{3}}$
(c) v (d) $\sqrt{3}\,v$

12 Water falls from a height of 60 m at the rate of 15 kg/s to operate a turbine. The losses due to frictional forces are 10% of energy. How much power is generated by the turbine? (Take, $g = 10\,\text{m/s}^2$) ➡ CBSE AIPMT 2008

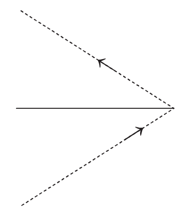
(a) 8.1 kW (b) 10.2 kW
(c) 12.3 kW (d) 7.0 kW

13 An intense stream of water of cross-sectional area A strikes a wall at an angle θ with the normal to the wall and returns back elastically. If the density of water is ρ and its velocity is v, then the force exerted in the wall will be



(a) $2Av\rho\cos\theta$ (b) $2Av^2\rho\cos\theta$
(c) $2Av^2\rho$ (d) $2Av\rho$

14 A mass of 100 g strikes the wall with speed 5 m/s at an angle as shown in figure and it rebounds with the same speed. If the contact time is 2×10^{-3} sec, then what is the force applied on the mass by the wall?



(a) $250\sqrt{3}$ N to right (b) 250 N to right
(c) $250\sqrt{3}$ N to left (d) 250 N to left

SESSION 1

1 As the road does not move at all, the work done by cycle on road is zero.

2 Work done in moving the object from $x = 0$ to $x = 6\,\text{m}$ is given by
W = area of rectangle + area of triangle

$$= 3\times3 + \frac{1}{2}\times3\times3 = 9 + 4.5 = 13.5\,\text{J}$$

3 Position vectors of the particles are
$\mathbf{r}_1 = -2\hat{\mathbf{i}} + 5\hat{\mathbf{j}}$ and $\mathbf{r}_2 = 4\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$
∴ Displacement of the particle,
$\Delta\mathbf{s} = \mathbf{r}_2 - \mathbf{r}_1$
$= 4\hat{\mathbf{j}} + 3\hat{\mathbf{k}} - (-2\hat{\mathbf{i}} + 5\hat{\mathbf{j}}) = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 3\hat{\mathbf{k}}$
Force on the particle, $\mathbf{F} = 4\mathbf{i} + 3\mathbf{j}$ N
∴ Work done, $W = \mathbf{F}\cdot\Delta\mathbf{s}$
$= (4\hat{\mathbf{i}} + 3\hat{\mathbf{j}})\cdot(2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 3\hat{\mathbf{k}}) = 8 - 3 = 5\,\text{J}$

4 Work done = Area under (F-d) graph

$$= 2\times(7-3) + \frac{1}{2}\times2\times(12-7)$$
$$= 8 + \frac{1}{2}\times10 = 8 + 5 = 13\,\text{J}$$

5 Given, force $\mathbf{F} = 3\hat{\mathbf{i}} + \hat{\mathbf{j}}$
$\mathbf{r}_1 = (2\hat{\mathbf{i}} + \hat{\mathbf{k}})\text{m}$
and $\mathbf{r}_2 = (4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}})\text{m}$
$\therefore \Delta s = \mathbf{r}_2 - \mathbf{r}_1 = (4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}}) - (2\hat{\mathbf{i}} + \hat{\mathbf{k}})$
$= (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}})\text{m}$
$\mathbf{W} = \mathbf{F}\cdot d\mathbf{s} = (3\hat{\mathbf{i}} + \hat{\mathbf{j}})\cdot(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}})$
$= 3\times2 + 3 + 0 = 6 + 3 = 9\,\text{J}$

6 Work done, $dW = F\cdot dx$

$$\Rightarrow W = \int_0^5 (7 - 2x + 3x^2)\,dx$$
$$\Rightarrow W = [7x - x^2 + x^3]_0^5$$
$$\therefore W = 7\times5 - (5)^2 + (5)^3 = 135\,\text{J}$$

7 The potential energy of a system increases, if work is done by the system against a conservative force.

$$-\Delta U = W_{\text{conservative force}}$$

8 Work done by all forces = Change in kinetic energy

$$= \frac{1}{2}m(v_f^2 - v_i^2) = \frac{1}{2}\times2(0-400)$$
$$= -400\,\text{J}$$

9 $v_1 = v_2 = v$ at a 30 ft from falling point.
Here, $m_1 = 2$ kg, $m_2 = 4$ kg.

Thus, $$\frac{K_1}{K_2} = \frac{\frac{1}{2}m_1v^2}{\frac{1}{2}m_2v^2}$$
$$= \frac{m_1}{m_2} = \frac{2}{4} = \frac{1}{2}$$

10 Increase in KE $= \frac{p^2}{2m}$

New momentum $= p + \frac{p}{5} = \frac{6p}{5}$

$$KE_f = \frac{\left(\frac{6}{5}p\right)^2}{2m} = \frac{36}{25}\frac{p^2}{2m}$$

$$\Delta KE = \frac{36}{25}\cdot\frac{p^2}{2m} - \frac{p^2}{2m} = \frac{11p^2}{25\times 2m}$$

Percentage increase = 44%

11 $F = ma = \frac{mv}{T}$

Instantaneous power $= Fv = mav$

$$= \frac{mv}{T}\cdot at = \frac{mv}{T}\cdot\frac{v}{T}\cdot t = \frac{mv^2}{T^2}\cdot t$$

12 $\because P = Fv = (ma)v = m\left(\frac{d^2x}{dt^2}\right)\left(\frac{dx}{dt}\right)$

Since, power is constant,

$\therefore \left(\frac{d^2x}{dt^2}\right)\left(\frac{dx}{dt}\right) = k$ or $\frac{d}{dt}\left(\frac{dx}{dt}\right)^2 = k$

or $\left(\frac{dx}{dt}\right)^2 = k_1 t$ or $\frac{dx}{dt} = \sqrt{k_1 t} = k_2 t^{1/2}$

or $x = k_3 t^{3/2} \Rightarrow t \propto x^{2/3}$

Hence, $\frac{dx}{dt} \propto t^{1/2} \propto x^{1/3}$

13 From the diagram, it is clear that from $x = -2$ m to $x = 0$, displacement of particle is along positive x-direction while force acting on the particle is along negative x-direction. Hence, work done is negative and is given by the area under (F-x) graph

$$W = -\frac{1}{2}\times 2\times 10 = -10\text{ J}$$

14 Given force is a constant force and work done by a constant force is always path independent.

15 As m is the mass per unit length, then rate of mass per second

$$= \frac{mx}{t} = mv$$

$\therefore$ Rate of KE $= \frac{1}{2}(mv)v^2 = \frac{1}{2}mv^3$

16 Net work done in sliding a body up to a height h on inclined plane

= Work done against gravitational force + Work done against frictional force

$\Rightarrow W = W_g + W_f$ ...(i)

but $W = 300$ J

$W_g = mgh = 2\times 10\times 10 = 200$ J

On, putting the value of W_g in Eq. (i), we get

$$300 = 200 + W_f$$

$\Rightarrow W_f = 300 - 200 = 100$ J

17 Given, the potential energy of a particle in a force field, $U = \frac{A}{r^2} - \frac{B}{r^1}$

For stable equilibrium, $F = -\frac{dU}{dr} = 0$

$$= \frac{dU}{dr} = -2Ar^{-3} + Br^{-2}$$

$$0 = -\frac{2A}{r^3} + \frac{B}{r^2} \quad \left(\text{As } \frac{-dU}{dr} = 0\right)$$

or $\frac{2A}{r} = B$

The distance of particle from the centre of the field $r = \frac{2A}{B}$

18 $W_1 = \frac{1}{2}kx_1^2 = \frac{1}{2}\times 5\times 10^3\times(5\times 10^{-2})^2$

$= 6.25$ J

$W_2 = \frac{1}{2}k(x_1 + x_2)^2$

$= \frac{1}{2}\times 5\times 10^3(5\times 10^{-2} + 5\times 10^{-2})^2$

$= 25$ J

Net work done $= W_2 - W_1$

$= 25 - 6.25 = 18.75$ J $= 18.75$ N-m

19 $k_P > k_Q$

In ***Case a*** Elongation (X) is same.

$\therefore W_P = \frac{1}{2}k_P X^2, W_Q = \frac{1}{2}k_Q X^2$

$\therefore W_P > W_Q$

In ***Case b*** Force of elongation is same.

$\therefore X_1 = \frac{F}{k_P}, X_2 = \frac{F}{k_Q}$

$$W_P = \frac{1}{2}k_1X_1^2 = \frac{1}{2}\frac{F^2}{k_P}$$

$$W_Q = \frac{1}{2}k_2X_2^2 = \frac{1}{2}\frac{F^2}{k_Q}$$

$[\because k_1 = k_P, k_2 = k_Q]$

$\therefore W_P < W_Q$

20 $W = \Delta K$

$\Rightarrow -Fs = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

$= 0 - \frac{1}{2}\times 10\times 10^{-3}\times 10000$

$\Rightarrow -F\times 10^{-2} = -\frac{1}{2}\times 10\times 10^{-3}\times 10^4$

$\therefore F = 5000$ N

21 Work done by spring + work done by gravity $= \Delta KE$

$W_s + W_g = 0 - 0$

$W_s = -W_g \Rightarrow W_s = -mgh$

22 Work done by gravitational force,

$W_g = mgh = 10^{-3}\times 10\times 1\times 10^3 = 10$ J

Now, from work-KE theorem, we have

$\Delta K = W_{\text{gravity}} + W_{\text{air resistance}}$

$\Rightarrow \frac{1}{2}\times mv^2 = mgh + W_{\text{air resistance}}$

$\Rightarrow W_{\text{air resistance}} = \frac{1}{2}mv^2 - mgh$

$= 10^{-3}\left(\frac{1}{2}\times 50\times 50 - 10\times 10^3\right)$

$= -8.75$ J

23 Work done by force $= \Delta KE = KE_f - KE_i$

$\Delta W + KE_i = KE_f$

$\int_{20}^{30} -0.1x\,dx + \frac{1}{2}\times 10\times 10^2 = KE_f$

$-0.1\left.\frac{x^2}{2}\right|_{20}^{30} + 500 = KE_f$

$-\frac{0.1}{2}(900 - 400) + 500 = KE_f$

$-0.1\times\frac{500}{2} + 500 = KE_f$

$-25 + 500 = KE_f = 475$ J

24 Change in potential energy

$\Delta PE = m_1g\left(h - \frac{h}{2}\right) = m_1g\frac{h}{2}$

$KE = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

$m_1v_1 = -m_2v_2 \Rightarrow v_2 = -\frac{m_1}{m_2}v_1$

From conservation principle, ΔPE = gain in KE

$m_1g\frac{h}{2} = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2\left(\frac{m_1}{m_2}v_1\right)^2$

$gh = v_1^2 + \frac{m_1}{m_2}v_1^2 = v_1^2 + \frac{1}{2}v_1^2 = \frac{3}{2}v_1^2$

$\Rightarrow v_1 = \sqrt{\frac{2gh}{3}}$

25 Let x be the extension in the spring. Applying conservation of energy,

$Mgx - \frac{1}{2}kx^2 = 0 - 0 \Rightarrow x = \frac{2Mg}{k}$

26 According to conservation of energy,

$\frac{1}{2}kL^2 = \frac{1}{2}Mv^2 \Rightarrow kL^2 = \frac{(Mv)^2}{M}$

$MkL^2 = p^2$ $[p = Mv]$

$\therefore p = L\sqrt{Mk}$

So, just after collision momentum of block (maximum) would be $L\sqrt{MK}$.

27 Initially body posseses only kinetic energy and after attaining a height, the kinetic energy is zero.

Therefore, loss of energy = KE − PE

$= \frac{1}{2}mv^2 - mgh$

$= \frac{1}{2}\times 1\times 400 - 1\times 18\times 10$

$= 200 - 180 = 20$ J

28 When string is released, then according to conservation of energy

$\frac{1}{2}mv^2 = mgh \Rightarrow v^2 = 2gh = 2gR$

Now, when it is at the bottom

$$T - mg\cos 0° = \frac{mv^2}{R}$$

$$T - mg = \frac{m}{R}(2gR)$$

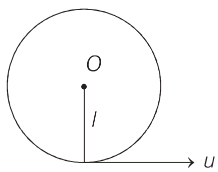
$$T = mg + 2mg = 3mg$$

29 When stone is at its lowest position, stone has only kinetic energy given by,

$K = \frac{1}{2}mu^2$

At the horizontal position, it has energy

$E = U + K' = \frac{1}{2}mu'^2 + mg\,l$



According to conservation of energy

$$K = E$$

$$\frac{1}{2}mu^2 = \frac{1}{2}mu'^2 + mgl$$

$$\frac{1}{2}mu'^2 = \frac{1}{2}mu^2 - mgl$$

$$u'^2 = u^2 - 2gl \Rightarrow u' = \sqrt{u^2 - 2gl}$$

So, the magnitude of change in velocity

$|\Delta u| = |u| = \sqrt{u'^2 + u^2 + 2uu'\cos 90°}$

$|\Delta u| = \sqrt{u'^2 + u^2 - 2} = \sqrt{2(u^2 - gl)}$

30 Equation of force, $mg - mg\cos\theta = \frac{mv^2}{l}$

$$\frac{v^2}{l} = g(1 - \cos\theta)$$

$\Rightarrow \quad v^2 = gl(1 - \cos\theta) \quad \quad \ldots$(i)

Applying law of conservation of energy

$$\frac{1}{2}mgl = \frac{1}{2}mv^2 + mgl(1 - \cos\theta)$$

$v^2 = gl - 2gl(1 - \cos\theta) \quad \quad \ldots$(ii)

On equating Eqs. (i) and (ii) and then solving, we get

$$\theta = \cos^{-1}\left(\frac{2}{3}\right)$$

From Eq. (ii), we get

$$v = \sqrt{\left(\frac{gl}{3}\right)} = \sqrt{\frac{9.8 \times 15}{3}} = 7 \text{ m/s}$$

31

$$T_{\text{top}} = \frac{mv^2}{l} - mg$$

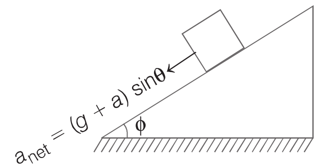
$$T_{\text{bottom}} = \frac{mv^2}{l} + mg$$

$$\frac{T_{\text{top}}}{T_{\text{bottom}}} = \frac{\left(\frac{v^2}{l} - g\right)}{\left(\frac{v^2}{l} + g\right)} = \frac{\left(\frac{40 \times 40}{4} - 10\right)}{\left(\frac{40 \times 40}{4} + 10\right)}$$

$$= \frac{400 - 10}{400 + 10} = \frac{390}{410} = \frac{39}{41}$$

32 Power $= Fv$

$\Rightarrow \quad F = \frac{P}{v}$



Given, $\quad P = 8 \times 10^5$ W, $v = 20$ m/s

$\Rightarrow \quad F = \frac{8 \times 10^5}{20} = 4 \times 10^4$ N

At constant speed, the forces acting on the train are in equilibrium. The force parallel to the hill is

$$F = R + (2 \times 10^5)g \times \frac{1}{50}$$

$$F = R + (2 \times 10^5) \times 9.8 \times \frac{1}{50}$$

$\Rightarrow \quad R = 4 \times 10^4 - 39200 = 800$ N

33 $Fv = \text{constant} = k \Rightarrow m\frac{dv}{dt}v = k$

$$\int v\,dv = \int \frac{k}{m}dt, \; \frac{v^2}{2} = \frac{k}{m}t$$

$\Rightarrow \quad v = \sqrt{\frac{2k}{m}}t, \; F = m\frac{dv}{dt}$

$$= m\sqrt{\frac{2k}{m}}\frac{1}{2}t^{-1/2}$$

$$= \sqrt{\frac{mk}{2}}t^{-1/2}$$

34 Given, Velocity of water $v = 2$ m/s

Mass per unit length of water in the pipe = 100 kg/m

So, power = (mass per unit length of water in pipe) $\times v^3$

$$= \frac{m}{l} \times v^3 = 100 \times 2 \times 2 \times 2 = 800 \text{ W}$$

35 Total initial energy $= \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2$

Total final energy

$= \frac{1}{2}m_2v_2^2 + \frac{1}{2}m_1v_1^2 + E$

From conservation of energy,

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + E$$

$$\Rightarrow \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 - E = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

36 It is given that mass of balls are same and collision is perfectly elastic ($e = 1$), so their velocities will be interchanged.

Thus, $v'_A = v_B = -0.3$ m/s,

$v'_B = v_A = 0.5$ m/s

37 We have, $p_1 + p_2 + p_3 = 0 \quad [\because p = mv]$

$\therefore \; 1 \times 12\hat{\mathbf{i}} + 2 \times 8\hat{\mathbf{j}} + p_3 = 0$

$\Rightarrow \quad 12\hat{\mathbf{i}} + 16\hat{\mathbf{j}} + p_3 = 0$

$\Rightarrow \quad p_3 = -(12\hat{\mathbf{i}} + 16\hat{\mathbf{j}})$

$\therefore \quad p_3 = \sqrt{(12)^2 + (16)^2}$

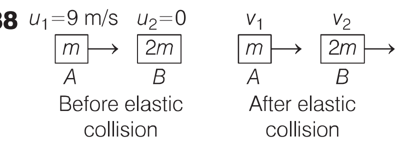
$= \sqrt{144 + 256}$

$= 20$ kg m/s

Now, $\quad p_3 = m_3v_3$

$\Rightarrow \quad m_3 = \frac{p_3}{v_3} = \frac{20}{4} = 5$ kg

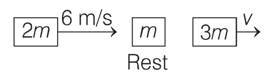
38



$$v_2 = \frac{2m_1u_1}{m_1 + m_2} = \frac{2 \times m \times 9}{m + 2m} = 6 \text{ m/s}$$

i.e. After elastic collision B strikes to C with velocity of 6 m/s.

Now, collision between B and C is perfectly inelastic.



By the law of conservation of momentum,

$2m \times 6 + 0 = 3m \times v_C$

$\Rightarrow \quad v_C = 4$ m/s

39 If A suffers on oblique elastic collision with a particle B, i.e. is at rest initially. If their massess are the same then after collision B remains at rest while A continuous to move in the original direction or they will move in mutually perpendicular directions.

40 Initial momentum $= p_i = 0$

Final momentum,

$p_f = 0 = mv\mathbf{i} + mv\mathbf{j} + p_3$

$\Rightarrow \quad p_3 = mv\sqrt{2}$

$$\text{Total KE} = \frac{p_3^2}{2 \times 2m} + \frac{1}{2}mv^2 + \frac{1}{2}mv^2$$

$$= \frac{2m^2v^2}{4m} + mv^2$$

$$= \frac{3mv^2}{2}$$

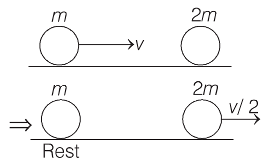
41 For inelastic collision between two spherical rigid bodies, total linear momentum is conserved.

42 Fraction of lost KE

$$= \frac{4x}{(1 + x)^2} = \frac{4 \times 4}{(1 + 4)^2}$$

$$= \frac{16}{25}$$

43 From conservation of linear momentum, we can see that velocity of $2\,m$ will become $\frac{v}{2}$ after collision (as mass is doubled)



Now, $e = \frac{\text{Relative velocity of separation}}{\text{Relative velocity of approach}}$

$= \frac{v/2}{v} = \frac{1}{2}$

44 Suppose a ball rebounds with speed v,

$v = \sqrt{2gh} = \sqrt{2 \times 10 \times 20}$

$= 20\text{ m/s}$

Energy of a ball just after rebound,

$E = \frac{1}{2}mv^2 = 200\text{ m}$

As, 50% of energy loses in collision means just before collision energy is 400 m.

According to law of conservation of energy, we have

$$\frac{1}{2}mv_0^2 + mgh = 400\text{ m}$$

$\Rightarrow \frac{1}{2}mv_0^2 + m \times 10 \times 20 = 400\text{ m}$

$\Rightarrow \qquad v_0 = 20\text{ m/s}$

45 By conservation of momentum,

$mv + M \times 0 = (m + M)v'$

Velocity of composite block

$v' = \left(\frac{m}{m+M}\right)v$

KE of composite block

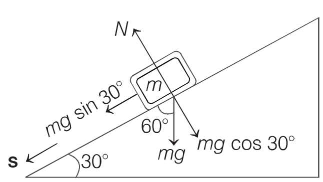
$= \frac{1}{2}(M + m)v^2$

$= \frac{1}{2}(M + m)\left(\frac{m}{M+m}\right)^2 v^2$

$= \frac{1}{2}mv^2\left(\frac{m}{m+M}\right)$

SESSION 2

1 We have, $s = \frac{1}{2}at^2 = \frac{1}{2} \times 2 \times 4 = 4\text{ m}$



$\therefore$ Work done by force of gravity

$W = mg \cdot s = mgs\cos 60°$

$= 10 \times 9.8 \times 4 \times \frac{1}{2} = 196\text{ J}$

2 Here, $m = 0.5\text{ kg}$, $v = ax^{3/2}$, where $a = 5\text{ms}^{-2}$, $W = ?$

Acceleration, $A = \frac{dv}{dt} = \frac{d}{dt}(ax^{3/2})$

$= a \times \frac{3}{2}x^{1/2}\frac{dx}{dt} = \frac{3a}{2}x^{1/2}(ax^{3/2})$

$A = \frac{3a^2}{2}x^2$

Force, $F = mA = 0.5 \times \frac{3a^2}{2}x^2 = \frac{3a^2}{4}x^2$

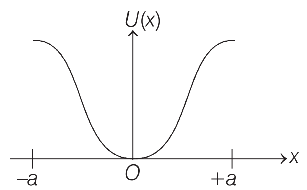
Work done,

$W = \int_0^2 F \cdot dx = \int_0^2 \frac{3a^2}{4}x^2\,dx$

$= \frac{3a^2}{4}\left[\frac{x^3}{3}\right]_0^2 = \frac{3}{4 \times 3}(5)^2[2^3 - 0]$

$W = 50\text{ J}$

3 The graph of $U(x)$ with x is as shown, potential energy is zero at $x = 0$ and maximum at $x = \pm a$.



Mechanical energy has fixed value $\frac{k}{2}$, kinetic energy has to be maximum at $x = 0$ and minimum at $x = \pm a$.

4 Given, mass of particle $m = 0.01$ kg.

Radius of circle along which particle is moving, $r = 6.4$ cm.

$\because$ Kinetic energy of particle,

$\text{KE} = 8 \times 10^{-4}\text{ J} \Rightarrow \frac{1}{2}mv^2 = 8 \times 10^{-4}\text{ J}$

$\Rightarrow \quad v^2 = \frac{16 \times 10^{-4}}{0.01} = 16 \times 10^{-2} \qquad \text{...(i)}$

As it is given that KE of particle is equal to 8×10^{-4} J by the end of second revolution after the beginning of motion of particle. It means, its initial velocity (u) is 0 m/s at this moment.

$\because$ By Newton's 3rd equation of motion,

$v^2 = u^2 + 2a_t s$

$\Rightarrow \quad v^2 = 2a_t s \quad \text{or} \quad v^2 = 2a_t(4\pi r)$

($\because$ particle covers 2 revolutions)

$\Rightarrow \quad a_t = \frac{v^2}{8\pi r} = \frac{16 \times 10^{-2}}{8 \times 3.14 \times 6.4 \times 10^{-2}}$

($\because$ from Eq. (i), $v^2 = 16 \times 10^{-2}$)

$\therefore \quad a_t = 0.1\text{ m/s}^2$

5 Work done

$= \frac{1}{2}mv^2 - \frac{1}{2}mu^2 = K_f - K_i$

$\therefore \qquad F \cdot dx = K_f - \frac{1}{2}mv_i^2$

$F \cdot dx = K_f - \frac{1}{2} \times 10 \times (10)^2$

$\Rightarrow \qquad F \cdot dx = K_f - 500$

$\Rightarrow \quad \int_{x=20}^{x=30}(-0.1)\,x\,dx = K_f - 500$

or $\quad -0.1\left[\frac{x^2}{2}\right]_{x=20}^{x=30} = K_f - 500$

$-0.1\left[\frac{(30)^2}{2} - \frac{(20)^2}{2}\right] = K_f - 500$

$\Rightarrow \quad K_f - 500 = -25$

$\Rightarrow \qquad K_f = 500 - 25 = 475\text{ J}$

6 According to the conservation of mechanical energy,

$(\text{TE})_{\text{initial}} = (\text{TE})_{\text{final}}$

$\Rightarrow \quad (\text{KE})_i + (\text{PE})_i = (\text{KE})_f + (\text{PE})_f$

$0 + mgh = \frac{1}{2}mv_A^2 + 0$

$\Rightarrow \quad gh = \frac{v_A^2}{2} \quad \text{or} \quad h = \frac{v_A^2}{2g} \qquad \text{...(i)}$

In order to complete the vertical circle, the velocity of the body at point A should be $v_A = v_{\min} = \sqrt{5gR}$

where, R is the radius of the body.

Here, $\quad R = \frac{AB}{2} = \frac{D}{2}$

$\Rightarrow \quad v_{\min} = v_A = \sqrt{\frac{5}{2}gD}$

Substituting the value of v_A in Eq. (i), we get

$$h = \frac{\left(\sqrt{\left(\frac{5}{2}gD\right)}\right)^2}{2g} = \frac{5gD}{2 \times 2g} = \frac{5}{4}D$$

7 According to question, we have

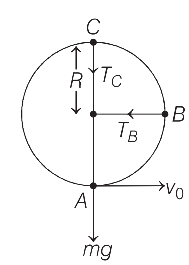
Let the tension at point A be T_A. So, force equation,

$T_A - mg = \frac{mv_c^2}{R}$

Energy at point $A = \frac{1}{2}mv_0^2 \qquad \text{...(i)}$

Energy at point C is

$\frac{1}{2}mv_c^2 + mg \times 2R \qquad \text{...(ii)}$



At point C,

$T_c + mg = \frac{mv_c^2}{R}$

To complete the loop $T_c \geq 0$

So, $mg = \frac{mv_c^2}{R}$

$\Rightarrow \quad v_c = \sqrt{gR}$ ...(iii)

From Eqs. (i) and (ii) by conservation of energy

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv_c^2 + 2mgR$$

$$\Rightarrow \quad mv_0^2 = mgR + 2mgR \times 2 \qquad [\because \; v_c = \sqrt{gR}]$$

$$\Rightarrow \quad v_0^2 = gR + 4gR$$

$$\Rightarrow \quad v_0 = \sqrt{5gR}$$

8 According to question, a body of mass 1 kg begins to move under the action of time dependent force,

$$\mathbf{F} = (2t\hat{\mathbf{i}} + 3t^2\hat{\mathbf{j}})\ \text{N}$$

where $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ are unit vectors along X and Y-axes.

$$\because \quad \mathbf{F} = m\mathbf{a} \Rightarrow \mathbf{a} = \frac{\mathbf{F}}{m}$$

$$\Rightarrow \quad \mathbf{a} = \frac{(2t\hat{\mathbf{i}} + 3t^2\hat{\mathbf{j}})}{1} \qquad (\because \; m = 1\ \text{kg})$$

$$\Rightarrow \quad \mathbf{a} = (2t\hat{\mathbf{i}} + 3t^2\hat{\mathbf{j}})\ \text{m/s}^2$$

$\because$ acceleration, $a = \frac{dv}{dt}$

$$\Rightarrow \quad dv = a\,dt \qquad \text{...(i)}$$

Integrating both sides, we get

$$\int dv = \int a\,dt = \int (2t\hat{\mathbf{i}} + 3t^2\hat{\mathbf{j}})\,dt$$

$$\mathbf{v} = t^2\hat{\mathbf{i}} + t^3\hat{\mathbf{j}}$$

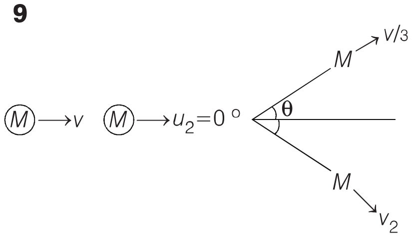
$\because$ Power developed by the force at the time t will be given as

$$P = \mathbf{F} \cdot \mathbf{v} = (2t\hat{\mathrm{i}} + 3t^2\hat{\mathrm{j}}) \cdot (t^2\hat{\mathrm{i}} + t^3\hat{\mathrm{j}})$$

$$= (2t.t^2 + 3t^2 \cdot t^3)$$

$$P = (2t^3 + 3t^5)\ \text{W}$$

9



According to law of conservation of kinetic energy, we have

$$\frac{1}{2}Mv^2 + 0 = \frac{1}{2}M\left(\frac{v}{3}\right)^2 + \frac{1}{2}Mv_2^2$$

$$\Rightarrow v^2 = \frac{v^2}{9} + v_2^2 \Rightarrow v^2 - \frac{v^2}{9} = v_2^2 \Rightarrow \frac{8v^2}{9}$$

Velocity of second block after collision,

$$v_2 = \frac{2\sqrt{2}}{3}v$$

10 Since, the collision mentioned is an elastic head-on collision. Thus, according to the law of conservation of linear momentum, we get

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

where, m_1 and m_2 are the masses of the two blocks, respectively and u_1 and u_2 are their initial velocities and v_1 and v_2 are their final velocities, respectively.

Here, $m_1 = m, m_2 = 4m$

$u_1 = v, u_2 = 0$

and $v_1 = 0$

$$mv + 4m \times 0 = 0 + 4mv_2$$

$$\Rightarrow \quad mv = 4mv_2$$

or $\quad v_2 = \frac{v}{4}$ ...(i)

As, the coefficient of restitution is given as,

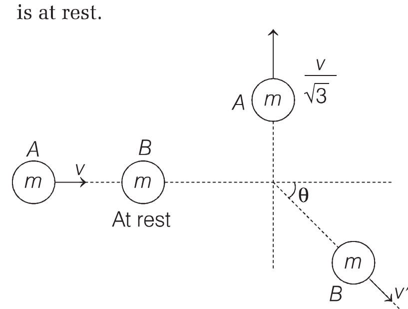
$$e = \frac{\text{relative velocity of separation after collision}}{\text{relative velocity of approach}}$$

$$= \frac{v_2 - v_1}{u_2 - u_1} = \frac{\frac{v}{4} - 0}{0 - v} \qquad \text{[from Eq. (i)]}$$

$$= \frac{1}{4}$$

$\therefore \; e = 0.25$

11 Let mass A moves with velocity v and collides inelastically with mass B, which is at rest.



According to problem mass A moves in a perpendicular direction and let the mass B moves at angle θ with the horizontal with velocity v.

Initial horizontal momentum of system (before collision) $= mv$ ...(i)

Final horizontal momentum of system (after collision) $= mv'\cos\theta$ ...(ii)

From the conservation of horizontal linear momentum

$$mv = mv'\cos\theta \Rightarrow v = v'\cos\theta \quad \text{...(iii)}$$

Initial vertical momentum of system (before collision) is zero.

Final vertical momentum of system $\frac{mv}{\sqrt{3}} - mv'\sin\theta$.

From the conservation of vertical linear momentum

$$\frac{mv}{\sqrt{3}} - mv'\sin\theta = 0 \Rightarrow \frac{v}{\sqrt{3}} = v'\sin\theta \quad \text{...(iv)}$$

On squaring and adding Eqs. (iii) and (iv), we get

$$v^2 + \frac{v^2}{3} = v'^2(\sin^2\theta + \cos^2\theta)$$

$$\Rightarrow \quad \frac{4v^2}{3} = v'^2 \Rightarrow v' = \frac{2}{\sqrt{3}}v$$

12 $P_{\text{generated}} = P_{\text{input}} \times \frac{90}{100}$

$$= \frac{mgh}{t} \times \frac{90}{100} = \frac{15 \times 10 \times 60}{1} \times \frac{90}{100}$$

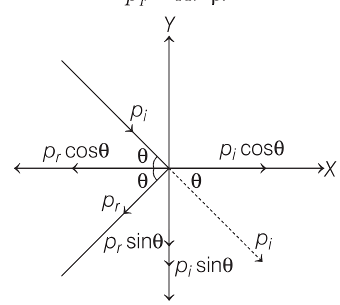
$= 8.1$ kW

13 Linear momentum of water striking per second to the wall

$$p_i = mv = Av\rho v = Av^2\rho,$$

similarly linear momentum of reflected water per second

$$p_r = Av^2\rho.$$



Now, making components of momentum along X and Y-axes. Change in momentum of water per second

$$= p_i\cos\theta + p_r\cos\theta = 2Av^2\rho\cos\theta$$

By definition of force, force exerted on the wall $= 2Av^2\rho\cos\theta$

14 Force = Rate of change of momentum

Initial momentum

$$\mathbf{p}_1 = mv\sin\theta\,\hat{\mathbf{i}} + mv\cos\theta\,\hat{\mathbf{j}}$$

Final momentum

$$\mathbf{p}_2 = -mv\sin\theta\,\hat{\mathbf{i}} + mv\cos\theta\,\hat{\mathbf{j}}$$

$$\therefore \quad \mathbf{F} = \frac{\Delta\mathbf{p}}{\Delta t} = \frac{-2mv\sin\theta}{2 \times 10^{-3}}$$

On substituting, $m = 0.1$ kg, $v = 5$ m/s, $\theta = 60°$

Force on the ball, $F = -250\sqrt{3}$ N to left

Negative sign indicates direction of the force.